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Surface roughness and transverse magnetic field dependence of the Hall coefficient and the magnetoresistance in thin metal films

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Defining an effective relaxation time which depends on the root mean square (rms) surface roughness and on the angle of incidence of electrons and then using the Boltzmann transport equation general expressions have been derived for the Hall coefficient and conductivity in thin metal films subjected to a transverse magnetic field. In the weak- and strong-field limits simple analytical equations have been proposed which reveal slight size effects in the Hall coefficient and in the magnetoresistance as well as a weak field dependence of these transport parameters in agreement with previous experiments. The theoretical predictions of the present model have been compared with those of the mean free path (mfp) method which constituted an extension of the Cottey model. In conclusion a correlation between the respective size parameters, A, in the present model and, μ, in the mfp method is proposed.

1. Introduction

It is well-known that the electrical conductivity is lower in thin films than in the bulk: a geometrical effect that in the past [1–3] has been generally interpreted in terms of the Fuchs Sondheimer (FS) theory [4]. However in the past few years more sophisticated size effect models for the film conductivity have been proposed [5–14]. Many investigations are concerned with extensions of the FS model [6, 7, 10–12] or the Cottey model [13, 14] which have been developed by introducing a specularity parameter, p, which depends explicitly both (either) on the angle of incidence, θ, of the conduction electron [7, 8, 10–14] and (or) on the rms surface roughness, r[5–9, 11–14]. Among the recent works [9–14] the Soffer–Cottey (SC) model [13, 14] based on the combination of the Cottey theory [15] with the Soffer model [7] leads to simple and analytical expressions for the conductivity of thin metal films and constitutes a convenient tool for an experimental determination of the surface roughness parameter in thin films.

But the interest in a size effect model may be revived if this model gives analytical formulas for various transport parameters and thus permits to test, with efficiency, the adequation of the present model with experiments. Hence this paper is devoted to the study of the influence of the transverse magnetic field on the transport properties of thin metal films in the framework of the combined SC model which considers the contribution of the rms surface roughness and of the angle of incidence to the distribution function of electrons [13]. Emphasis is made to derive analytical expressions for the Hall coefficient and for the film conductivity in the limit of vanishingly small magnetic field and of very strong magnetic field.

2. General theoretical treatment

2.1. The general solution of the Boltzmann equation

In terms of the SC model a relaxation time $\tau(\theta, r)$ describes the simultaneous background scattering and the electron scattering at the external surfaces respectively located at $z = 0$ and $z = d$.

$$\tau(\theta, r) = \tau_0(1 + A\cos^2\theta|\cos\theta|)^{-1} \quad (1)$$

where τ_0 is the background relaxation time. The parameter A contains the ratio of the rms surface roughness, r, to the wavelength, λ_c, associated to the carrier and the ratio, k, of the film thickness, d, to the background mean free path, λ_0:

$$A = \frac{1}{k}\left(\frac{4\pi r}{\lambda_c}\right)^2 \quad (2)$$

In this condition and for a geometry related to a metal film placed in an electric field $\boldsymbol{E}$ (E_x, E_y, 0) and a transverse magnetic field $\boldsymbol{H}(0, 0, H)$ the Boltzmann equation in isothermal conditions can be expressed as

$$\frac{f^1}{\tau(\theta, r)} - \frac{eH}{m}\left(v_y\frac{\partial f^1}{\partial v_x} - v_x\frac{\partial f^1}{\partial v_y}\right) = \frac{e}{m}\left(E_x\frac{\partial f^0}{\partial v_x} + E_y\frac{\partial f^0}{\partial v_y}\right) \quad (3)$$

where e is the absolute change of an electron, m is the effective electron mass, f^1 is the deviation from the equilibrium distribution function, f^0, caused by the external electrical and magnetic fields and v_x and v_y are components of the free electron velocity $\boldsymbol{v}$ (absolute value v).

Equation 3 is generally solved by following a

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procedure previously proposed by Sondheimer [16] which consists

(a) firstly, to let

$$f^1 = (v_x C_1 + v_y C_2)\frac{\partial f^0}{\partial v} \qquad (4)$$

where C_1 and C_2 are functions which do not depend explicitly on v_x and v_y.

(b) Secondly, to introduce the complex quantities

$$g = C_1 - jC_2 \qquad (5a)$$

$$F = E_x - jE_y \qquad (5b)$$

in order to rewrite the system of the two following equations

$$\frac{C_1}{\tau(\theta, r)} + \frac{eH}{m}C_2 = \frac{e}{mv}E_x \qquad (6a)$$

$$\frac{C_2}{\tau(\theta, r)} - \frac{eH}{m}C_1 = \frac{e}{mv}E_y \qquad (6b)$$

which results from substitution of Equation 4 into Equation 1 in the final compact form

$$\frac{g}{\tau(\theta, r)} + j\frac{v}{r_B}g = \frac{e}{mv}F \qquad (7)$$

Examination of Equation 7 gives the immediate general solution

$$g = \frac{e}{mv} \times \frac{(E_x - jE_y)\left[\frac{1}{\tau_0}\left(1 + A\cos^2\theta|\cos\theta| - j\frac{v\tau_0}{r_B}\right)\right]}{\left(\frac{1}{\tau_0}\right)^2\left[(1 + A\cos^2\theta|\cos\theta|)^2 + \left(\frac{v\tau_0}{r_B}\right)^2\right]} \qquad (8)$$

where r_B is the radius of the Larmor orbit of an electron moving in a magnetic field of magnitude H, i.e.

$$r_B = \frac{mv}{eH} \qquad (9)$$

Defining the field parameter

$$\alpha = \lambda_0/r_B \qquad (10)$$

the functions C_1 and C_2 are finally found to be expressed as

$$C_1 = \frac{M}{D(\theta)}\{E_x(1 + A\cos^2\theta|\cos\theta|) - \alpha E_y\} \qquad (11a)$$

$$C_2 = \frac{M}{D(\theta)}\{E_y(1 + A\cos^2\theta|\cos\theta|) + \alpha E_x\} \qquad (11b)$$

where

$$M = e\tau_0/mv \qquad (12)$$

and where for convenience $D(\theta)$ is a function defined as

$$D(\theta) = (1 + A\cos^2\theta|\cos\theta|)^2 + \alpha^2 \qquad (13)$$

2.2. General expressions for the current density

Since the current density along the i direction ($i = x, y$) is obtained by summing the i-component of the velocity of all the conduction electron, the components of the current density are readily found to be

$$J_x = -2e\left(\frac{m}{h}\right)^3 \iiint C_1 v_x^2 \frac{\partial f^0}{\mathrm{d}v}\, d^3v \qquad (14a)$$

$$J_y = -2e\left(\frac{m}{h}\right)^3 \iiint C_2 v_y^2 \frac{\partial f^0}{\partial v}\, d^3v \qquad (14b)$$

The above integrals are to be evaluated by introducing polar coordinates (v, θ, Ψ) in the $\boldsymbol{v}$ space with $v_z = v\cos\theta$ and using Equations 11a and 11b. After carrying the integration over v and Ψ one obtains the final equations for the current density.

$$J_x = \tfrac{3}{4}\sigma_0\{E_x I_1 - \alpha E_y I_2\} \qquad (15a)$$

$$J_y = \tfrac{3}{4}\sigma_0\{E_y I_1 + \alpha E_x I_2\} \qquad (15b)$$

where I_1 and I_2 represent integrals over the variable θ

$$I_1 = \int_0^\pi D^{-1}(\theta)\{1 + A\cos^2\theta|\cos\theta|)\}\sin^3\theta\,\mathrm{d}\theta \qquad (16)$$

$$I_2 = \int_0^\pi \sin^3\theta\, D^{-1}(\theta)\,\mathrm{d}\theta \qquad (17)$$

After transforming the integration variable from θ to $u = \cos\theta$ the electron current density in the x direction and in the y-direction can be written in the alternative forms

$$J_x = \tfrac{3}{2}\sigma_0\{E_x\mathscr{A} - \alpha E_y\mathscr{B}\} \qquad (18a)$$

$$J_y = \tfrac{3}{2}\sigma_0\{E_y\mathscr{A} + \alpha E_x\mathscr{B}\} \qquad (18b)$$

with

$$\mathscr{A} = \int_0^1 \frac{(1 + Au^3)(1 - u^2)}{(1 + Au^3)^2 + \alpha^2}\,\mathrm{d}u \qquad (19)$$

$$\mathscr{B} = \int_0^1 \frac{1 - u^2}{(1 + Au^3)^2 + \alpha^2}\,\mathrm{d}u \qquad (20)$$

2.3. General expressions for the transport parameters

Since the current is for the geometry of the model confined to the x axis, the Hall coefficient, R_{Hf}, and the electrical conductivity, σ_f, of the metal film are respectively defined by

$$R_{Hf} = \left.\frac{E_y}{HJ_x}\right|_{J_y=0} \qquad (21)$$

$$\sigma_f = \left.\frac{J_x}{E_x}\right|_{J_y=0} \qquad (22)$$

Thus the reduced Hall coefficient and electrical conductivity are readily found to be

$$R_{Hf}/R_{Ho} = \frac{2}{3}\frac{\mathscr{B}}{\mathscr{A}^2 + \alpha^2\mathscr{B}^2} \qquad (23)$$

$$\sigma_f/\sigma_0 = \frac{3}{2}\frac{\mathscr{A}}{\mathscr{A}^2 + \alpha^2\mathscr{B}^2} \qquad (24)$$

where R_{H_0} and σ_0 are respectively the Hall coefficient and the electrical conductivity of the bulk metal.

Unfortunately Equations 19 and 20 cannot be, in general, expressed in terms of simple or tabulated integrals and numerical integration becomes necessary. However, it is possible to obtain analytical expressions for the transport parameters in the special cases of low and strong magnetic fields. The results of these calculations are expected to be sufficient to illustrate the effects of the rms surface roughness and of the angular dependence and to undertake a comprehensive comparison with the predictions of theoretical models [16–18] which assume that the electron scattering at the film surfaces can be described by a constant specularity parameter.

3. Analytical expressions for the transport parameters

At this point it should be noticed that published results [13, 14] on the film conductivity in absence of magnetic field have revealed that

(1) In general, the conditions of validity of the combined (SC) model extend without ambiguity to about $r/\lambda_c \simeq 0.15$.

(2) The experimental determination of the surface roughness parameter in metal films remains very easy for small A.

Hence for these reasons we restrict our calculations to metal films with relatively smooth surfaces ($r/\lambda_c < 0.2$) or to relatively thick films ($k > 0.4$), namely for films with small values of the size parameter A even if the analytical expressions derived later in the special case of strong magnetic fields applied over a larger range of values of A. The conditions retained in this section are effectively met in many experiments [19–26].

3.1. The limit of small magnetic fields

The functions (and Equations 19 and 20) can be analytically evaluated by expanding the integrals firstly in ascending power of the field parameter α and secondly in ascending power of Au^3. Retaining only terms of power two for α and A the integration yields

$$\mathscr{A} \simeq \frac{2}{3} - \frac{A}{12} + \frac{2A^2}{63} - \alpha^2\left\{\frac{2}{3} - \frac{A}{4} + \frac{4A^2}{21}\right\},\quad A < 1, \alpha \ll 1 \tag{25}$$

$$\mathscr{B} \simeq \frac{2}{3} - \frac{A}{6} + \frac{2A^2}{21} - \alpha^2\left\{\frac{2}{3} - \frac{A}{3} + \frac{20A^2}{63}\right\},\quad A < 1, \alpha \ll 1 \tag{26}$$

The transport parameters, σ_f, and R_{Hf} can then be easily evaluated using the respective general Equations 24 and 23.

3.2. The limit of strong magnetic fields

In order to evaluate R_{Hf} and σ_f we now expand Equations 19 and 20 in ascending power of $(1 + Au^3)^2/\alpha^2$. The expressions for $\mathscr{A}$ and $\mathscr{B}$ are considerably simplified and after carrying out the integrations one obtains the following analytical equations

$$\mathscr{A} \simeq \frac{2}{3\alpha^2}\left\{1 + \frac{A}{8} - \frac{1}{\alpha^2}\left(1 + \frac{3A}{8} + \frac{A^2}{7} + \frac{A^3}{40}\right)\right\},\quad (A + 1)/\alpha \ll 1 \tag{27}$$

$$\mathscr{B} \simeq \frac{2}{3\alpha^2}\left\{1 - \frac{1}{\alpha^2}\left(1 + \frac{A}{4} + \frac{A^2}{21}\right)\right\},\quad (A + 1)/\alpha \ll 1 \tag{28}$$

The appropriate numerical values for R_{Hf} and σ_f are obtained as before by substituting $\mathscr{A}$ and $\mathscr{B}$ from Equations 27 and 28 into the general formulas.

Moreover examination of Equation 27 and 28 reveals that in the limits of small A and very strong magnetic fields σ_f and R_{Hf} can be respectively approximated by the following simple forms

$$\sigma_f/\sigma_0 \simeq \left\{1 + \frac{A}{8}\right\}^{-1},\quad A \ll 1, \alpha \gg 1 \tag{29}$$

$$R_{Hf}/R_{H_0} \simeq 1,\quad A \ll 1, \alpha \gg 1 \tag{30}$$

4. Discussion

4.1. Presentation of theoretical results

Before embarking on the presentation of theoretical results let us state that we shall primarily concern ourselves here with a comparison of the present model with two theoretical models, namely the Sondheimer model [16] and the mean free path (mfp) model previously derived by Tellier *et al.* [17] and based as in the Cottey model [15] on the definition of an effective relaxation time. Thus in this section we collect the numerical solutions of Equations 23 and 24 which have been evaluated for different values of the parameters, k, r/λ_c and α using the appropriate approximate expressions for $\mathscr{A}$ and $\mathscr{B}$.

In Fig. 1 plots of the conductivity ratio, σ_0/σ_f, against the reduced roughness, r/λ_c, are shown for different values of the field parameter, α, and for

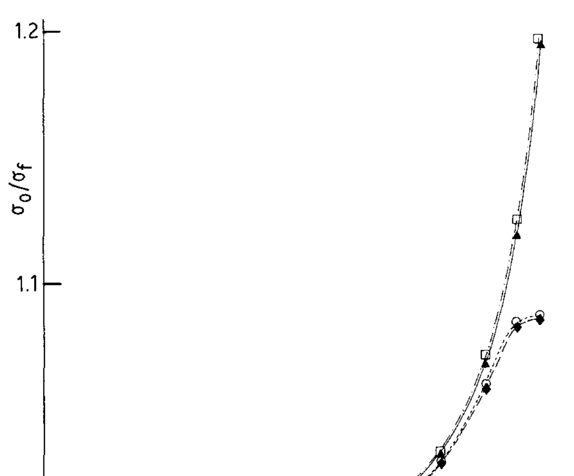

Figure 1 The surface roughness variation of the conductivity ratio, σ_0/σ_f for different values of the field parameter, α (□) $\alpha = 100$, (▲) $\alpha = 40$, (○) $\alpha = 0.1$, (◆) $\alpha = 0.01$.

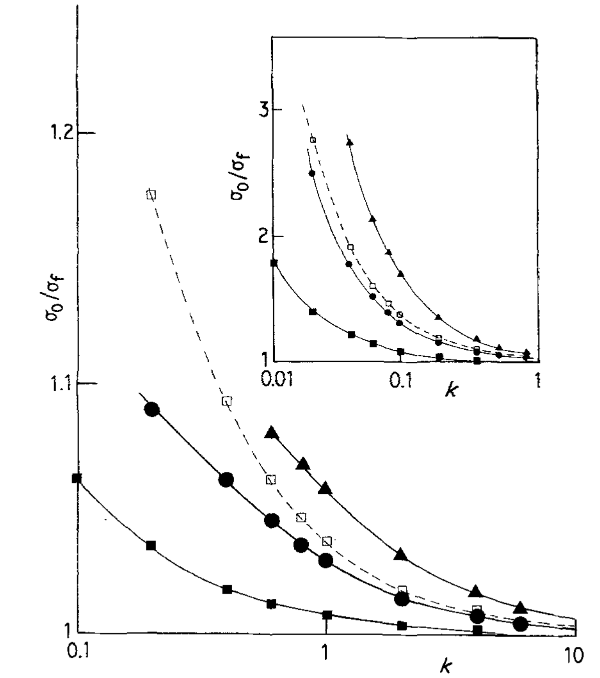

Figure 2 Variations in the conductivity ratio, σ_0/σ_f, with the reduced thickness, k, in the weak-field limit ($\alpha = 0.04$). Full curves are for the present model. The dotted curve is the result of the mfp method for $p = 0.9$. In the inset are shown the corresponding variations in the conductivity ratio in the strong-field limit ($\alpha = 40$). (▲) $r/\lambda_c = 0.06$ (●) $r/\lambda_c = 0.04$, (■) $r/\lambda_c = 0.02$.

$k = 1$. These curves indicate that in a relative independency of the magnetic field strength the film conductivity decreases markedly with increasing rms surface roughness. Representative variations of the conductivity ratio as a function of the reduced thickness are shown in Fig. 2 for different values of the field parameter and of the rms surface roughness. Here the predicted behaviour agrees closely with the Sondheimer and the mfp models [16, 17], the σ_0/σ_f ratio decreases rapidly and approaches unity as the reduced thickness, k, tends to infinity.

For relatively large thicknesses and for relatively smooth surfaces the numerical evaluation of the conductivity ratio reveals that the effect of a magnetic field remains of slight importance with respect to the consequence of the limitation of the mean free path and of the rms surface roughness (see Fig. 1 for example). Thus a convenient scheme for displaying the effect of the field variation is through the magnetoresistance defined by the ratio

$$\frac{\Delta\varrho_f}{\varrho_{f,0}} = \frac{\varrho_f(H) - \varrho_f(0)}{\varrho_f(0)} \tag{31}$$

Representative results are shown in Fig. 3 for two values of r/λ_c and for $k = 1$. From these curves we see that the magnetoresistance $\Delta\varrho_f/\varrho_{f,0}$ is small and is a slowly varying function of the field parameter α which will not exceed 2% in the strong-field limit. However let us note that the transverse magnetoresistance is extremely sensitive to peculiarities of the Fermi surface and that a model based on the quasi-free electron approach will lead to theoretical values of magnetoresistance which are orders of magnitude smaller than experimental values. Obviously here we restrict ourselves to an additional magnetoresistance depending on sample dimensions and on surface irregularities which will be observable in some metals [27]. Indeed an interesting feature of the limiting magnetoresistance is its dependence on the size parameter A. The anomalous large magnetoresistance one can observe in intermediate magnetic fields is the result of deviations from the validity conditions imposed in the derivation of the approximate expressions for $\mathscr{A}$ and $\mathscr{B}$. In reality $\Delta\varrho_f/\varrho_{f,0}$ will never attain such magnitudes, and more realistic plots of the expected magnetoresistance in the intermediate field region are drawn (dotted curves) in Fig. 3 where a monotonically increase of the magnetoresistance with the field parameter α is assumed.

From Equations 23 and 25–28, we see that the Hall coefficient of film reduces to the value of the bulk material with increasing magnetic fields. In fact in the case of slight surface effects small deviations from the bulk behaviour are always expected and the influence of the transverse magnetic field remains too small (see Equations 23 and 24 for example) to give rise to specific size effects even in vanishing magnetic fields. Numerical results displayed in Tables I and II agree well with these observations. The dependence of the Hall coefficient on the magnetic field appears as very weak provided $A \ll 1$. Moreover the size effect in the variation of the Hall coefficient with either the film thickness or the rms surface roughness is much lower

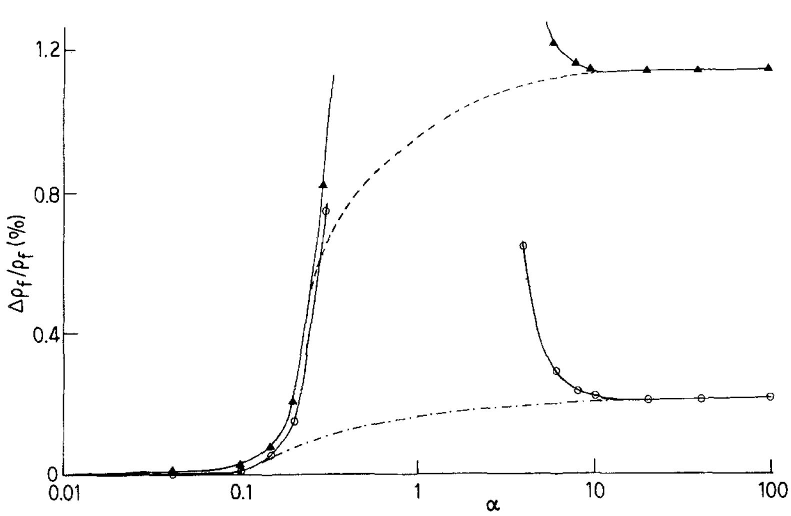

Figure 3 Field dependence of the magnetoresistance for $k = 1$ and for different values of the rms surface roughness (▲) $r/\lambda_c = 0.06$, (○) $r/\lambda_c = 0.04$. The dotted curves correspond to the expected field dependence of the magnetoresistance in the intermediate field region.

TABLE I Field and surface roughness variations of the Hall coefficient ratio R_{Hf}/R_{Ho} choosing a typical value of 1 for the reduced thickness, k. Omitted values correspond to deviations from the conditions of applicability of the approximate equations

r/λ_c	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 10$	$\alpha = 100$
0.01	1.00001	1.00011	1.00010	1.00000
0.04	1.00236	1.00242	1.00013	1.00000
0.06	1.01378	1.01370	1.00024	1.00000
0.1	–	–	1.00107	1.0001

TABLE II The field variation of the Hall coefficient ratio, R_{Hf}/R_{Ho} for moderately rough film surfaces ($r/\lambda_c = 0.04$): influence of the film thickness

k	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 10$	$\alpha = 100$
0.1	–	–	1.00257	1.00002
0.4	1.01741	1.01731	1.00027	1.00000
1	1.00236	1.00242	1.00013	1.00000
4	1.00013	1.00023	1.00010	1.00000
10	1.00002	1.00012	1.00010	1.00000

in magnitude than the variation in the electrical conductivity. Effectively in the weak- and strong-field limits the relative variations in R_{Hf} with A are less than 2% for A in the range 0.001–0.5.

4.2. Comparison with other models

At the first sight the present model leads to rough agreement with previous theoretical models [16, 17] which treated the size effect in the Hall coefficient by using a constant specularity parameter p. Indeed in the weak-field limit the Hall coefficient, R_{Hf}, in thin films is greater than the bulk value R_{Ho}, and the behaviour of the film approaches that of a bulk specimen in the strong-field limit. To make the comparison more significant we have tabulated (Table III) the numerical values for the Hall coefficient as evaluated in terms of the present model and in terms of the mfp model. The value of 0.9 has been selected for the constant specularity parameter p since it has been found to correspond nearly to a reduced surface roughness of about 0.04 when the electron reaches the film surface at an incidence angle of about 45°. Actually, the discrepancies between the mfp model and the present model are frequently fairly small (Table III) for the Hall coefficient. In particular we do not observe that the major feature of the SC model is a diminution of the overall size effect in this transport parameter as previously established for the electrical conductivity in the absence of a magnetic field [13]. But since in the α, A or μ ranges investigated here the Hall coefficient of films are restricted to values close to the bulk value it remains difficult to draw firm conclusions. Thus in view of the features just discussed some tabulations concerned with the film conductivity have been inserted (Table IV) to allow a comparison between the orders of magnitude and between the essential features displayed by the two models. Inspection of Table IV reveals at once that in the weak- and strong-field limits the SC model does not yield the same conductivity as in the mfp model: a decrease in the overall size effect is still caused by incorporating angular and rms surface roughness effect just as predicted in a previous study when $H = 0$ [13].

Thus at this point it appears to be interesting to consider precisely what might happen if the specularity parameter can no longer be considered as constant. This is best undertaken by comparing in details the analytical expressions for the function $\mathscr{A}$ and $\mathscr{B}$ as derived respectively in terms of the mfp model and the SC model. In the framework of the mfp model and in the weak-field limit the function $\mathscr{A}^*$ and $\mathscr{B}^*$ are expanded in the forms [18]

$$\mathscr{A}^* \simeq \sum_{i=0}^{n} \gamma_i^* \mu^{-i} + \alpha^2 \sum_{i=0}^{n} a_i^* \mu^{-i} \tag{32}$$

$$\mathscr{B}^* \simeq \sum_{i=0}^{n} B_i^* \mu^{-i} + \alpha^2 \sum_{i=0}^{n} b_i^* \mu^{-i} \tag{33}$$

TABLE III The Hall coefficient ratio R_{Hf}/R_{Ho} for different values of the reduced thickness. In the strong field limit we are restricted to relatively small k since for $k > 0.4$ inaccuracies of the numerical work of the mfp method causes the development of oscillations of the Hall coefficient ratio

k	Weak-field limit: $\alpha = 0.04$		k	Strong-field limit: $\alpha = 40$	
	Present model ($r/\lambda_c = 0.04$)	mfp model ($p = 0.9$)		Present model ($r/\lambda_c = 0.04$)	mfp model ($p = 0.9$)
0.4	1.01740	1.00300	0.01	1.01432	1.00362
1	1.00236	1.00054	0.04	1.00081	1.00023
4	1.00013	0.99996	0.1	1.00013	1.00056
10	1.00002	0.99893	0.4	1.00001	0.99997

TABLE IV The thickness variation of the conductivity ratio σ_0/σ_f as evaluated in the framework of the mfp method and of the combined SC model

k	Weak-field limit: $\alpha = 0.04$		k	Strong-field limit: $\alpha = 40$	
	Present model ($r/\lambda_c = 0.04$)	mfp model ($p = 0.9$)		Present model ($r/\lambda_c = 0.04$)	mfp model ($p = 0.9$)
0.4	1.06382	1.09042	0.01	3.99184	4.72959
1	1.02939	1.03693	0.04	1.7864	1.93697
4	1.00777	1.00932	0.1	1.31555	1.37522
10	1.00314	1.00347	0.4	1.07895	1.10467

TABLE V The coefficients of expansion is given in the framework of the mfp model [17] and of the present model

i	mfp model				SC model	
	γ_i^*	B_i^*	γ_i^*/a_i^*	B_i^*/b_i^*	γ_i/a_i	B_i/b_i
0	$\frac{2}{3}$	$\frac{2}{3}$	-1	-1	-1	-1
1	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{2}$
2	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{3}{10}$	$-\frac{1}{6}$	$-\frac{3}{10}$

where μ is the Cottey size effect parameter generally defined as

$$\mu = k \left\{ \varrho n \left(\frac{1}{p} \right) \right\}^{-1} \tag{34}$$

Values of the low-order coefficients ($i = 0, 1, 2$) are listed in Table V.

Turning to Equation 25 and 26, one obtains the alternative forms for $\mathscr{A}$ and $\mathscr{B}$

$$\mathscr{A} \simeq \sum_{i=0}^{n} \gamma_i A^i + \alpha^2 \sum_{i=0}^{n} a_i A^i \tag{35}$$

$$\mathscr{B} \simeq \sum_{i=0}^{n} B_i A^i + \alpha^2 \sum_{i=0}^{n} b_i A^i \tag{36}$$

For i varying from 0–2 the coefficients just defined in the above equations are also displayed in Table V. From this table we see that the values of the ratios γ_i/a_i and b_i/B_i are the same as the values of ratios appearing in Equations 32 and 33. This analogy is not surprising since values of coefficient ratios do not depend on the assumptions about the specularity parameter but are primarily determined by the procedure chosen to solve the transport Boltzmann equation. But Table V also displays a more interesting result. Until the size effect cannot significantly influence the transport properties ($A < 1$ or $\mu > 1$) a clear correspondence can be established between the size parameter A and μ

$$A \rightarrow 3\mu^{-1} \tag{37}$$

which is just the result obtained by Tellier [14] when the film is subjected only to the action of an electric field.

But the interest in this correspondence is revived if its range of applicability extends to moderate and strong fields. Remember that according to previously published results [16, 17] the mfp analysis must yield simple limiting equations for the transport parameters in strong magnetic field

$$R_{\mathrm{Hf}}^*/R_{\mathrm{Ho}} \simeq 1 \tag{38}$$

$$\sigma_0/\sigma_{\mathrm{f}}^* \simeq 1 + \frac{3}{8\mu} \tag{39}$$

Now considering Equation 29 we can conclude that in the special case of slight size effects the new limiting expressions for the transport parameters can be easily derived from expressions previously presented in the mfp model since it seems sufficient to replace the size parameter μ by the SC term $3/A$.

4.3. Comparison with experimental works

It should be pointed out that in this section we are essentially concerned with experimental works [22–26] which were performed at room-temperature on well annealed films. Indeed for thoroughly annealed film it is reasonable to suppose that a reordering of the film surface occurs [1, 3, 28–31] causing the size parameter A to take relatively small values. Moreover we restrict ourselves to temperatures and thicknesses in which quantum size effects and Shubnikov de Haas oscillations [22] do not occur.

Hoffman and Frankl [22] measured at 300 K the Hall coefficient and the magnetoresistance of well-ordered bismuth films. At room temperature, the magnetoresistance of relatively thin films was found to be practically independent of field. Also in the work on bismuth films by Ineou *et al.* [24] a quite similar feature was reported for the magnetoresistance which, at room temperature, remained smaller than 4%. Data on magnetoresistance in silver–gold films [26] showed small $\Delta\varrho/\varrho$ values which did not exceed 3%. These behaviours agree with the slight magnetoresistance effect predicted by the present model.

Tabulated results on the Hall coefficient (Tables I and II) give some evidence for a weak field dependence of the Hall coefficient for $A < 1$. The experimental behaviours of R_{Hf} as a function of field H in silver films [23] and in silver-gold films [26] conformed nearly to this theoretical feature.

Measurement of the Hall coefficient in thin gold films by Jeppesen [25] revealed no appreciable variation in the Hall coefficient for films which varied in thickness from 3–200 nm. Also Viard *et al.* [23] as well as Bhattacharya and Bhattacharya [26] observed slight size effects in the Hall coefficient of silver films. Hoffman and Frankl [22] found that in thin bismuth films the Hall coefficient and the magnetoresistance varied smoothly with the film thickness.

We thus observe that some predictions of the present model concerned with the field dependence of the transport parameters and with the size effects in the Hall coefficient and the magnetoresistance are confirmed by various experimental works.

5. Conclusion

In the presence of a transverse magnetic field new general equations are derived for the electrical conductivity and the Hall coefficient taking into account that the specularity parameter depends on the angle of incidence of the carriers and on the rms surface roughness. In the weak- and strong-field limits simple expressions for these transport parameters can be obtained which are found to be convenient to reveal some interesting features of the Hall coefficient and the magnetoresistance. It is predicted that the magnetoresistance is weakly dependent on the field and the size parameter A, in good agreement with published results. In the case of slight size effects ($A < 1$) in films the Hall coefficient follows nearly a similar behaviour. Comparison with the theoretical predictions of the mfp model which assumes that the specularity parameter, p, appearing in the size parameter μ is constant, gives some evidence for a simple correlation between the parameters A and μ provided $A < 1$.

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